Two Hot, Two Cold

Concepts

- Graph Scatterplots
- Analyze and graph linear equations
- Compute and model slope
- Derive and apply a conversion equation

Materials

- TI-84 Plus
- 2 EasyTemp™ Probes or 2 EasyLinks™ with temperature probes
- EasyData™ Application
- Several cups of water with varying temperatures
- Ice and boiling water

Overview

In this experiment, you will collect data in both Celsius and Fahrenheit temperatures using two temperatures probes in the same cups of water. Based on the data collected, you will develop an equation to convert Celsius temperatures to Fahrenheit temperatures.

Overview

How do we measure temperatures? In almost all countries of the world, the Celsius scale (formerly called the centigrade scale) is used in everyday life, science, and industry. This scale sets the freezing temperature of water at 0 and the boiling temperature at 100. The distance between these two points is divided into 100 equal intervals called degrees.

The United States uses Fahrenheit scale. This scale employs a smaller degree unit than the Celsius scale and its freezing point is set to a different temperature. For the temperatures we commonly use and observe, Celsius readings are lower than Fahrenheit readings. You have probably noticed this if you have seen a thermometer that has both Celsius and Fahrenheit markings or if you have driven by signs at banks and other businesses that display time and dual temperatures.

Setup

1. Each group will need 5 cups with water of varying temperatures.
   - One cup of water should be room temperature or fresh out of the tap.
   - One cup of water should be very cold with many solid ice cubes in the cup.

- One cup of water should be cool with only a few cubes of ice that have just melted.
- One cup of water should be considerably warmer than room temperature.
- One cup of water should be either boiling water or very close to boiling.

2. The order the readings are taken is not important. An efficient way to test each cup is to have a few samples in one area of the room and let students carry one or two samples at a time from that area to the area where their group is working.

- After taking those readings, the samples can be returned. This will allow you to keep the water boiling for each temperature reading.
- Run water through a coffee pot for hot water samples, and keep ice in a small cooler for the cold water samples.

3. Have students work in groups of 3 or 4. Have them assign each group member a task. For this lab, suggest the student jobs:

- Materials/setup person who sets up the EasyTemp™ sensors and TI-84 Plus calculators, and holds probes in sample during the activity.
- Two Tech people who each operates EasyData™ APP and a TI-84 Plus, one for Celsius readings and one for Fahrenheit readings.
- Data recorder who reads and records the temperature readings for each sample on the worksheet chart.
- Runner who brings samples to the group, holds the cup to avoid spillage during the activity, and returns samples after readings are taken.

4. Link two EasyTemp™ probes to two different TI-84 Plus calculators.

- You may also use the two EasyLinks™ with two of the older temperature probes or a combination of these two setups.
- Refer to the Figure 1.

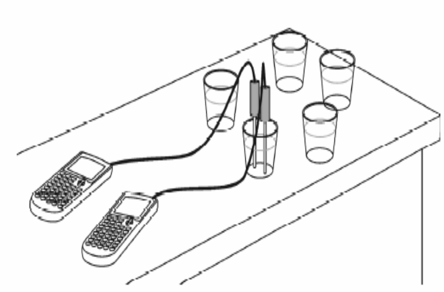


Figure 1

Data Collection

1. When you connect the EasyTemp™ probe to your TI-84 Plus, the EasyData™ APP will launch automatically.
    - Connecting the EasyLink™ to your TI-84 Plus, and connecting an older temperature sensor to an older temperature sensor to the other end of the EasyLink™ will work just as well.
2. The EasyData™ information screen is displayed for about 3 seconds followed by the main screen.
    - The EasyData™ APP identifies the temperature sensor.
    - The main screen of EasyData™ will display the current temperature across the top of the screen in degrees Celsius (Figure 2).

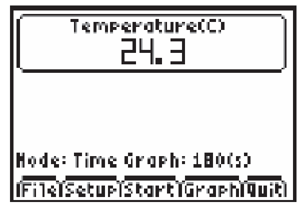


Figure 2

3. Press Y= to access the **File** menu, and select **1:New** by pressing 1.
    - Or, since **1:New** is highlighted, you can press ENTER.
    - This resets the program, and clears out old data (Figure 3).

Figure 3

4. Leave one of the calculators with the Celsius setting but the other calculator will need to be changed to Fahrenheit.
    - Press WINDOW to select **Setup**, and choose **1:Temp** from the menu displayed (Figure 4).

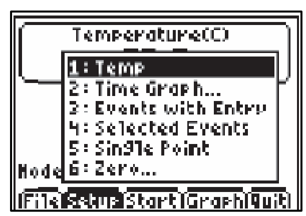


Figure 4

5. From the next screen, press WINDOW to select **Units** (Figure 5).

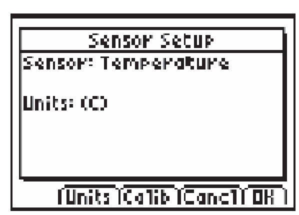


Figure 5

6. Select **2:(F)** from the menu to change from Celsius to Fahrenheit (Figure 6).

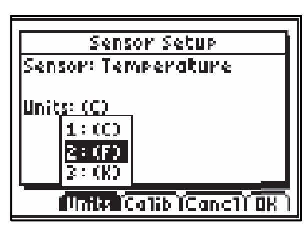


Figure 6

7. When the screen appears confirming that your choice has been accepted, select **OK** (Figure 7).

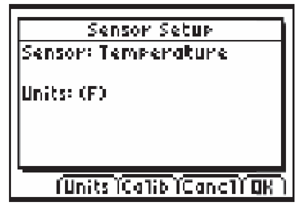


Figure 7

8. You will be returned to the main screen of the APP.

- Select **Setup** on both calculators, and select **4:Selected Events** (Figure 8).

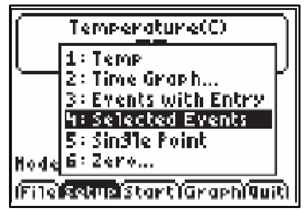


Figure 8

9. You will be taken to a screen that displays the temperature reading in real time at the top of the screen.

- See Figure 9 and 10.

10. To begin colleting data, position the two probes next to each other but not touching.

- Select **Start**.
- Let the first reading be the room temperature
- To record the temperature of the first reading, select **Keep** (Figure 11).

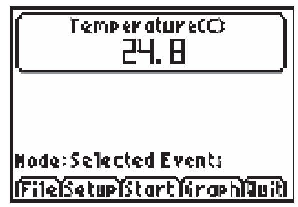


Figure 9

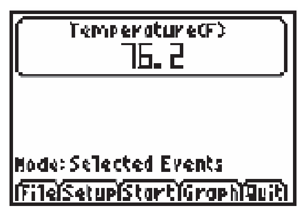


Figure 10

11. Next, place both probes in the first cup.

- If the first cup contains the room temperature water, the reading will probably be close to, if not the same as, the air temperature.
- When it stabilizes, select **Keep**.
- The **Selected Events** feature will keep track of which reading you are on, increasing by one each time you select **Keep**.

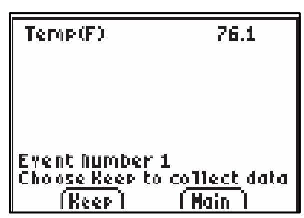


Figure 11

12. Repeat the procedure until you have collected the six readings. This includes the air temperature and the five cups of water.

- Have students record the readings in the table on their worksheet.
- Make sure the students understand the order in which they collect the data is not important.
- The important thing is to keep the two probes close together so they are measuring the same temperature.

- Select **Keep** at the same time on both calculators after the temperature readings have become relatively stable (Figure 12).

13. With each recorded value, a new data point will be displayed on the graph.

- When finished, select **Stop**.
- The graph of all the data points will be displayed.
- As this point you can use the right and left arrow keys to view the coordinates of the points (Figure 13).

14. You will **NOT** analyze the data from within the APP.

- Select **Main** and then Quit to exit the APP.

15. The new screen will inform you that your data are in lists one and two.

- **L1** contains the numbers 1-6.
- The temperatures are stored in **L2** (Figure 14).
- The Celsius and Fahrenheit readings are stored in **L2** on two separate calculators.
- The lists need to be in both calculators so that the Celsius data is in **L1** and the Fahrenheit is in **L2**.
- This allows you to examine the relationship between them.

Note: The lists could be renamed CELSI and FAHRN at this point, but only a few students would get the rename the lists. If you wait until the data is linked, every student will get the experience of creating and naming a list.

16. On the calculator that took the readings in the Celsius scale, press STAT, and select **1:Edit** to see the lists displayed.

- We need to replace **L1** with the values in **L2** (Figure 15).

17. Position the cursor so the name **L1** is highlighted.

- Press 2nd 2 to access **L2**.
- You will see **L2** at the bottom of the screen.
- Press ENTER (Figure 16).

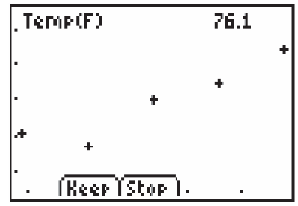


Figure 12

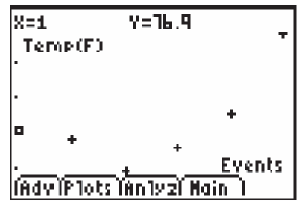
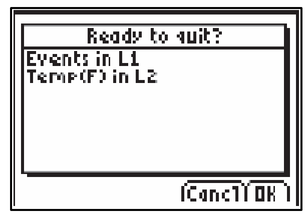


Figure 13



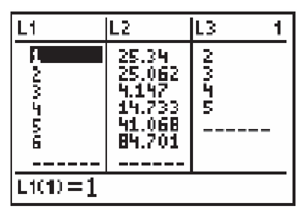


Figure 15

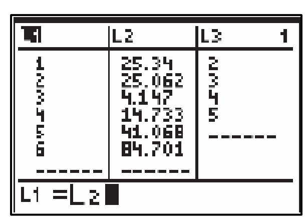


Figure 16

18. **L1** should fill in with the data from **L2** (Figure 17).

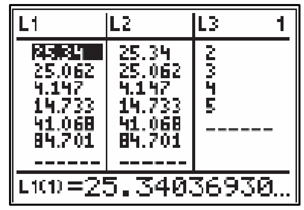


Figure 17

19. Link the two calculators together, and pass **L1** from the Celsius calculator to the Fahrenheit calculator and then **L2** from the Fahrenheit calculator to the Celsius calculator.
    - In both cases, because the receiving calculator already has data in the target list, your screen will look like Figure 18.
    - Select **2:Overwrite**.

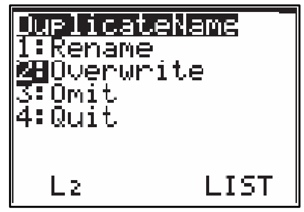


Figure 18

20. When the list is sent successfully, you will receive confirmation (Figure 19).

Figure 19

21. Press [STAT], and select **1:Edit** to see the lists displayed on both calculators (Figure 20).
    - Link both **L1** and **L2** to all students' calculators.

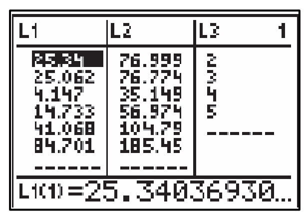
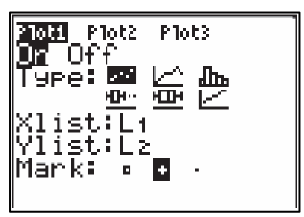


Figure 20

Data Analysis

1. Set up a scatterplot with temperature in degrees Celsius as the independent variable (Xlist) and the corresponding temperature in degrees Fahrenheit as the dependent variable (Ylist).
    - See Figure 21.



2. Press [ZOOM], and select **9:ZoomStat** to see the graph of the scatterplot.
    - When [TRACE] is selected, you may have trouble scrolling through the points.
    - When a scatterplot is traced using the right arrow key, the points are scrolled through in the order they were entered in the data list of the independent variable.

- Often this is the order in which they appear on the screen from the left to right, but that is not what happened in this scatterplot.
- The right arrow will allow you to scroll through the points in the order they are listed in **L1** and **L2** regardless of where they appear on the screen (Figure 22).

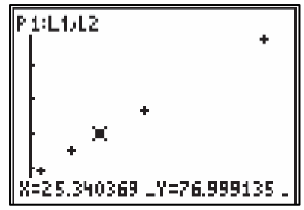


Figure 22

3. Sort the lists so that the data points are in order from smallest to largest.
    - Use the calculator to sort the list for you.
    - Press [STAT], and select **2:SortA(** from the menu.
    - This will sort the list in ascending order (Figure 23).

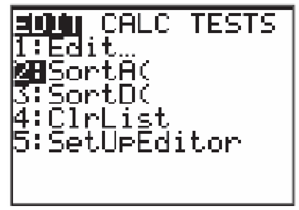


Figure 23

4. This takes you to the home screen.
    - If you enter **L1**, the calculator will arrange the number in the list one in order, but it will leave the numbers in **L2** alone.
    - Because the numbers in **L2** are related to the numbers in **L1**, the entire row needs to be carried along with the lead entry from **L1**.
    - To do this, type **SortA(L1, L2).**
    - Press [ENTER] to execute the command (Figure 24).

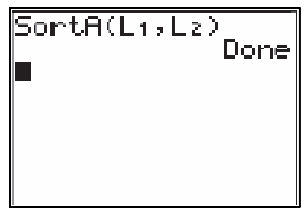


Figure 24

5. Press [STAT], and select **1:Edit…**
    - Your data has been sorted.
    - Notice that the elements in L1 have been listed in ascending order, as have their corresponding values in L2 (Figure 25).

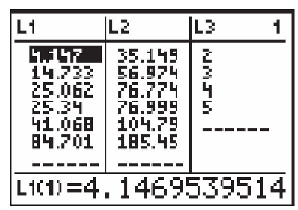


Figure 25

6. Now the points can be traced in order from left to right (Figure 26).

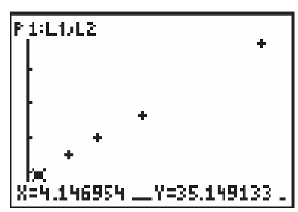


Figure 26

7. Next, find the trend line or the line of best fit. If your students have enough experience, you can discuss with them the different ways to find the regression equation.
    - One method is to estimate the slope and **Y-**intercept, and enter it into **Y1**.
    - Next, check to see how closely it matches the points and adjust the values until you are satisfied with the fit.

- This can be done manually or using the **Manual-Fit** option under the [STAT] [▸] **CALC** menu.
- Another method is to choose two ordered pairs on the graph and calculate the slope.
- Then, using the slope and **Y**-intercept (or any other point), find the equation of the line, graph the line, and see how well it fits the data points.
- Using the built-in linear regression features of the calculator is a quick and accurate method.

Why would it be best to avoid using the built-in regression in this case? Consider the purpose of this activity. If the purpose is to find the function relating Celsius and Fahrenheit temperatures, then looking up the formula in a book is as quick and accurate as using the regression feature. Our purpose is more for the students to learn linearity, that linear data has a constant slope, and that if data has a constant slope it can be modeled with a line. Also, as students adjust the parameters to find a better fit, they cement in their minds the definitions of **m** and **b** and what effects these parameters have on the equation of a line. Learning a few keystrokes to find a regression line and calling that “the answer” limits the power of the activity to teach students about the equation of a line.

8. The calculator can help with the computation of the slope.

- After tracing to the first point, press [2nd] [MODE] to [QUIT] and return to the home screen.
- Press [X,T,Θ,n] [STO▸] [ALPHA] **A** [ENTER].
- This will store the **X**-value from the point you last traced on the graph screen to the variable **A.**
- Repeat this procedure to store the **Y**-value in **B**.
- Press [ALPHA] **Y** [STO▸] [ALPHA] **B** [ENTER] (Figure 27).

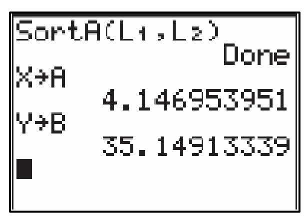


Figure 27

9. Press [GRAPH] [TRACE] [▸] to move to the last point on the right.

- Once again, notice the **X-** and **Y-**values displayed at the bottom of the screen.

10. Repeat the procedure to store these values in **C** and **D**.

- Press [2nd] [MODE] to access [QUIT], and return to the home screen.
- Press [X,T,Θ,*n*] [STO▸] [ALPHA] **C** [ENTER].
- This will store the **X**-value from the last point to the variable **C**.
- Next press [ALPHA] **Y** [STO▸] [ALPHA] **D** [ENTER] (Figure 28).

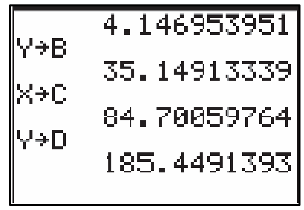


Figure 28

11. Using the slope definition, have the calculator find the slope and store the value in **M** as shown in the screen.

- Be sure to enclose both the numerator and denominator in parentheses.
- The keystroke sequence is [(]**D-** [ALPHA] B [)] / [(] [ALPHA] **C-** [ALPHA] **A** [)] [STO▸] [ALPHA] **M** [ENTER] (Figure 29)

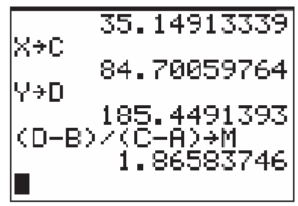


Figure 29

12. Go to the [Y=] window, and press [ALPHA] **M** [X,T,Θ,*n*] to type in **MX** behind **Y1** (Figure 30).

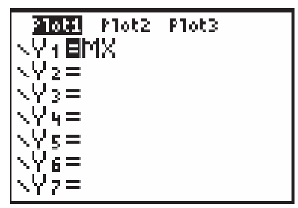


Figure 30

13. Press [GRAPH] to see how closely this line fits the points.

- In the example shown, it looks like the slope is correct since the line is parallel to an imaginary line through the points.
- The vertical position of the line needs to be moved up by adjusting the **Y**-intercept (Figure 31).

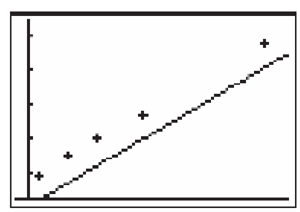


Figure 31

14. Determine how much your line is *below* where it needs to be.

- This great opportunity to discuss the importance of the **Y**-intercept.
- Be sure your students understand that with the present equation in **Y1**, the **Y**-intercept is 0.
- Ask the students if anyone knows what 0 degrees Celsius would be in Fahrenheit.

- Although you could figure out the **Y**-intercept mathematically, use this piece of general knowledge and add it to **Y1** (Figure 32).

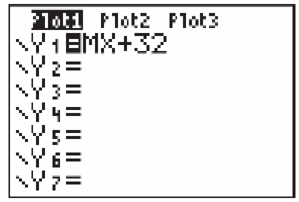


Figure 32

15. Press [GRAPH] to see how closely this fits the points.

- In the example shown, it looks like a great fit (Figure 33).

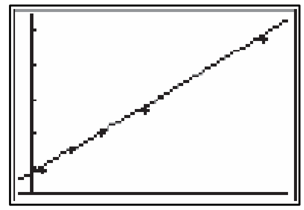


Figure 33

16. Look up the formula to convert Celsius to Fahrenheit, and enter it into **Y2**.

- Use the left arrow key to highlight the slash icon in front of **Y2**.
- Repeatedly press [ENTER] until you see the symbol shown in Figure 34.
- It is the symbol with the ball and the small line to the left of the ball.

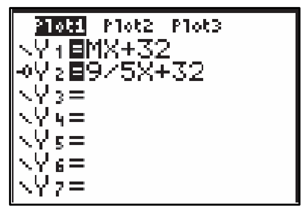


Figure 34

17. Press [GRAPH].

- **Y1** is graphed normally.
- A circular cursor traces the leading edge of the graph of **Y2** and helps emphasize how close the lines are to each other (Figure 35).

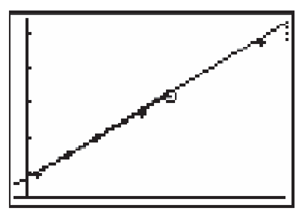


Figure 35

18. Pressing [2nd] [GRAPH] to access [TABLE] will allow you to see how close your regression equation is to the formula.

- A numerical comparison rather than just the visual comparison on the graph screen can confirm that the two lines are very close to being the same line (Figure 36).

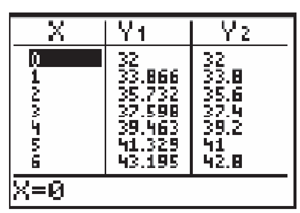


Figure 36

19. Press [2nd] [WINDOW] to access [TBLSET].

- The defaults on the table are to start at zero, to count by one, and to Automatically fill in all the values.
- With these settings, it could take a while to scroll and find specific values (Figure 37).

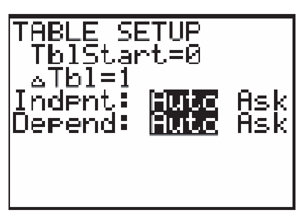


Figure 37

20. To take more control over what numbers the table displays, change the **Indpnt:** to **Ask** instead of **Auto**.

- Leave the **Depend:** set on **Auto**.
- Use the arrow keys to position the cursor on the word **Ask**, and press [ENTER] (Figure 38).

Figure 38

21. Press [2nd] [GRAPH] to access .

- You will be taken to a blank table.
- Type zero.
- It will be entered beside the **X=** at the bottom of the screen (Figure 39).

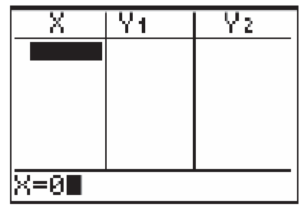


Figure 39

22. Pressing [ENTER] will fill in both **Y1** and **Y2**.

- Remind the students that they entered the **Y**-intercept from their knowledge of the fact that 0 degrees Celsius is 32 degrees Fahrenheit.
- So, this exact match is not an indicator for all points (Figure 40).

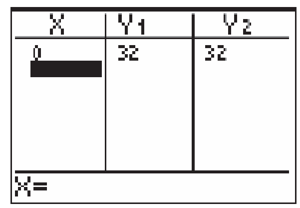


Figure 40

23. Another common set of values is the boiling point of water. Celsius is 100 degrees and Fahrenheit is 212.

- Have the students type in 100 beside the **X=**, and compare the values in **Y1** to the value in **Y2** (Figure 41).

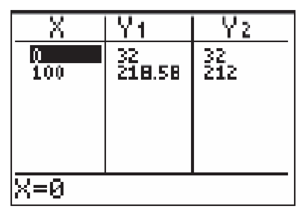


Figure 41

24. The goal of this lesson is to understand and highlight the algebra involved in building a mathematical model for a linear set of data.

- Additionally, the chance to help students become more familiar with the Celsius scale is a result.
- Here's a short poem that also helps:
  - *30's* hot and *20's* nice: *10* is cold and *0's* ice!
- Try saying it, then add these numbers to the **X**-values in the table to see their matching Fahrenheit values (Figure 42).

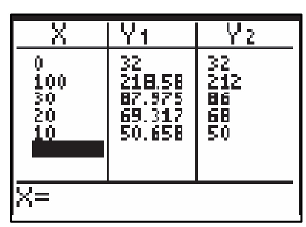


Figure 42

25. An extension to this activity it to let the calculator find a regression equation for the data, put in **Y3**, and then compare the various methods used in both the graph and the table views.

26. To accomplish this, press [STAT] [▸] [CALC], and choose **4:LinReg(ax+b)**.

- You will be taken to the home screen to enter the list names and where you want the equation pasted.
- The keystroke sequence is [2nd] [1] [,] [2nd] [2] [,] [VARS] [▸] to access **Y-VARS**, and select **1:Function**.
- From the list displayed, select **3:Y3** (Figure 43).

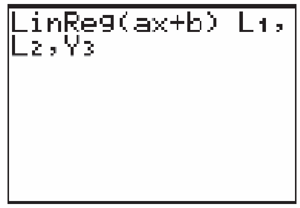


Figure 43

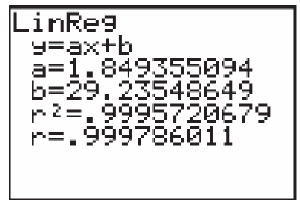


Figure 44

27. Press [ENTER] to execute the command (Figure 44).

28. Press [Y=], and use the arrow keys to highlight the slash icon in front of **Y3**.

- Repeatedly press [ENTER] until you see the symbol shown in Figure 45.
- The symbol will look like a ball with a small line to the left of the ball.

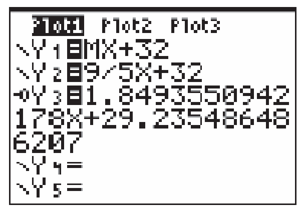


Figure 45

29. A circular cursor traces the leading edge of the graph of **Y3**, and it helps emphasize how close the lines are (Figure 46).

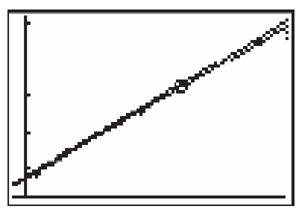


Figure 46

30. It is not unusual for the equation a student gets on his or her own to be closer to the actual formula than the regression equation found by the calculator. Many students think the calculator will always do a better job than they can do. This is a great chance to show them that they can be more accurate that a calculator.

- See Figures 47 and 48.

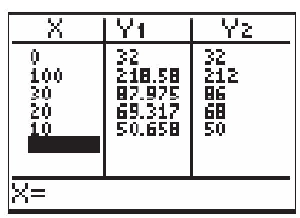


Figure 47

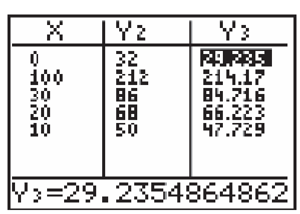


Figure 48

Two Hot, Two Cold

Student Activity Sheet

Student Name:________________________________

In this experiment, you will collect data in both Celsius and Fahrenheit temperatures using two temperatures probes in the same cups of water. Based on the data collected, you will develop and test a mathematical relationship between these two temperature scales, and explore the relationship between them. Your teacher will outline the procedure for you.

1. Fill in the table with the data you collected.

Cup of Water	ºC	ºF
Air room temperature		
Tap water		
Ice water (lots of ice still present)		
Cool water, ice has melted		
Warm water		
Very hot/boiling water		

2. Why is the order in which you collect the data not important?_____________________

3. You put the Celsius readings in **L1** and the Fahrenheit readings in **L2**. Why did you throw away the numbers 1-6 that were originally in **L1** after running the EasyData ™ APP?

____

4. You used two of these data points to find the slope of the regression equation. Would you get the same answer as if you used two different points? Explain._________________

____

5. What was your regression equation?____

6. The definition of the slope of a line is the change in **Y** divided by the change in **X**. For this problem, that would be a change in Fahrenheit divided by the change in Celsius. Use your own words to state what that means in respect to this problem. _______________

____

7. What was the regression equation the calculator found?__________________________

8. What is the reasonable explanation for why the two regression equations are different from each other and also different from the formula?___________________________

____

9. If your equation is in **Y1**, the formula is **Y2**, and the calculator's regression equation is in **Y3**, use the **Ask** feature on the table of the calculator and fill in this chart for the given temperatures.

X°C	**Y1**	**Y2**	**Y3**
5			
15			
25			
35			
60			
85			

Worksheet Answers

1. Answers will vary.
2. Because we were comparing the two temperature readings to each other.
3. They had nothing to do with how the two temperature readings were related.
4. Maybe not exactly the same but VERY close.
5. Answers will vary, but they should still be close to $y = 1.8x + 32$.
6. For every 1 degree change in Celsius, the Fahrenheit changes by 1.8 degrees.
7. Answers will vary, but they should be close to $y = 1.8x + 32$.
8. Small amount of human error and/or calibration of temperature probes.
9. **Y2** is from the formula; other answers will vary.

X°C	**Y2**
5	41
15	59
25	77
35	95
60	140
85	185